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A Mathematical Analysis of Akan Adinkra Symmetry

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Introduction

The Akan 'Adinkra' symbols are pictograms and ideograms which have come to be well known as cultural-linguistic symbols of the Akan people of Ghana. There are hundreds of these symbols. They represent ideas such as the nature of the universe, political beliefs and organization, social, economic and ethical values, aesthetics as well as ideas relating to family (see G. F. K. Arthur's book *Cloth as Metaphor*). These symbols not only represent specific ideas and proverbs but are also related to other forms of Akan art such as brick laying (building houses) metal crafts, weaving and wood carving, all of which can display versions of these symbols in their work.

As to the origin of these symbols, there has so far not been a conclusive or definitive answer. Some have suggested that the symbols derive from Islamic talismans. Others have pointed out that this cannot be the case because of the block-printing technique used by the Akan (which has not changed in centuries) is different from the writing brush and stick method used in Islamic inscriptions (G. F. K. Arthur, 2001, p22-25). What is known for sure is that the Bron, the Gyaman and the Denkyira, three of the old centers of Akan culture, were involved with weaving before the Asante rose to power. It is also said that the name 'Adinkra' comes from a Gyaman king who wore the cloth. For more information on the origin of these symbols, see G. F. K. Arthur's book.

While Rattray published 51 of these symbols in his book *Religion and Art in Ashanti*, G. F. K Arthur's book contains 719 of these symbols, although some are variations of the same. Out of these hundreds of symbols there are many with interesting symmetrical properties that I became interested in studying. Also, unlike Rattray's 51 symbols, some of the 719 symbols given by G. F. K. Arthur contain ideas from modern times.

In this paper, I look at the symmetrical properties of some Adinkra symbols. Although not all of these symbols have symmetry, there is a subset of them that do possess symmetries that fit a well studied group known as the 'Dihedral group' that describes symmetries found in art as well as in some geometric shapes in nature. We shall be looking at this some more, in the following pages.

Basic Group Theory and The Dihedral Group

i) **Brief sketch of Group Theory**

Group theory is a branch of mathematics that looks at mathematical structures with certain specific properties. Most common groups are finite structures that have internal consistency. For a mathematical structure to be classified as a group, it has to satisfy certain conditions. There is an operation which when performed on one member of the group gives a result that points to another member of the group. There are also the concepts of an 'identity', the presence of an 'inverse' for each element and another concept in mathematics called 'associativity'.

There are many, many different kinds of structures that can be classified as groups, but as a simple example let us use the numbers called 'integers', which most people are familiar with. The integers are numbers like ... -3, -2, -1, 0, 1, 2, 3, ... The dots before and after the list of numbers says that the list of numbers goes on forever in each direction, positive and negative.

As far as the integers are concerned, the 'identity' is the number zero. What this means is that any number added to zero will return that number. So $1 + 0 = 1$; $-5 + 0 = -5$; $69 + 0 = 69$ etc. All that the identity does is return the original element when the group operation is applied. In the case of integers, the group operation is addition so when you take any other element of the group of integers together with the identity (zero) and you apply the group operation (addition) to these two elements you get the non-identity element (i.e. the other number, not zero). That is what the examples above showed.

The inverse of an element in a group is any other element in the group which, when the group operation is applied to both elements gives the identity. What does this mean?

Let us return to our group example, the integers! Let us choose any number, say 6. What we want is to choose another number so that when that number is added to 6 will give us zero. Yes, you got it! What we need is to choose -6, so that $6 + (-6) = 0$. In this case, we say that for our group (integers) with group operation addition, the inverse of the number 6 is -6 and the inverse of the number -6 is 6, because when both numbers are added together, we get the 'identity', which we have already said is....zero.

The final requirement for a group is this idea called 'associativity'. All that this means can be demonstrated with an example:

$1 + (2 + 3) = (1 + 2) + 3 = 6$

Someone who looks at this example may say that we have not done anything. But this is incorrect. We use BIDMAS (some people learnt this as 'BODMAS') to evaluate the above express. 'BIDMAS' stands for 'Brackets, Index, Division, Multiplication, Addition, Subtraction', while in BODMAS, the 'O' stands for 'order', as in, what something is the power of. So the two terms are essentially the same. They show which operations to work before which.

What the example above shows is that the arithmetic operation 'addition' satisfies this condition called 'associativity' when applied to integers. This does not always work with every arithmetic operation. Let us test this with the same numbers:

$2 \times (3 \times 4) = (2 \times 3) \times 4 = 24$

So we can say that the arithmetic operation 'multiplication' is also 'associative' when applied to integers. What about subtraction?

$2 - (3 – 4) = 3$ which is <u>not</u> equal to $(2 – 3) – 4 = -5$

So we can say that the arithmetic operation 'subtraction' is not associative when applied to integers. The same can be said of the arithmetic operation 'division', which shall now be shown:

$2 \div (3 \div 4)$ gives 2.666 (recurring) while $(2 \div 3) \div 4$ gives 0.1666 (recurring) so the two expressions are different depending on where the brackets are placed.

So associativity means “within an expression containing two or more occurrences in a row of the same associative operator, the order in which the operations are performed does not matter as long as the sequence of the operands is not changed.” (Wikipedia)

That ends this brief treatment of basic group theory. Now let us relate group theory ideas to the Dihedral.

ii) **The Dihedral Group**

The dihedral group is the group of symmetries of a regular polygon that include reflections and rotations. The dihedral group is a type of 'symmetry group'.

If you consider an equilateral triangle (triangle with all three sides of equal length, and all three angles of equal size), and if you were to label the three corners A, B, and C, you can imagine rotating the triangle clockwise so that A takes the position of B, B takes the position of C and C takes the position of A. This can be seen as one rotation. The angle of rotation will be 60 degrees, since each angle in an equilateral triangle has that angle.

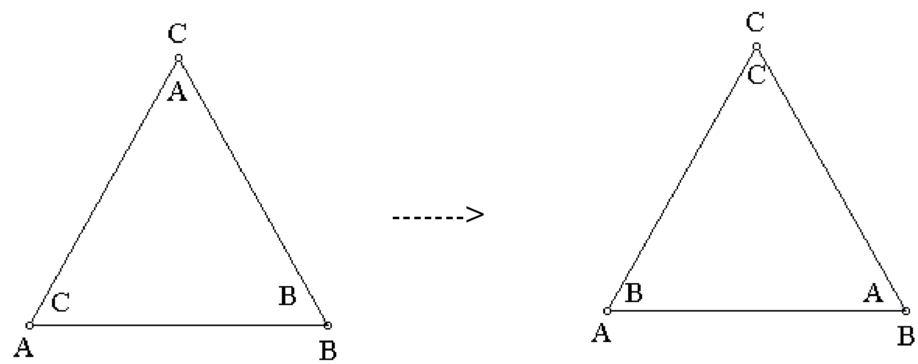


Fig 1 – Rotating a triangle 60° clockwise

Likewise, one can see how the triangle can be rotated again and again so that the corner labelled A will return to its original position. If one were to count the number of rotations needed to do this, the answer will be 3 (you can try it if you like).

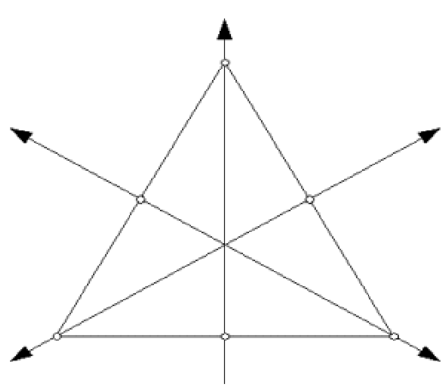


Fig 2 – Lines of symmetry of a triangle

One can also draw lines of symmetry as shown in the figure above. The triangle has three of them, while the square has four of them. If we were to carry out the symmetry operations on a square whose 4 corners are labelled (A, B, C and D) and which has 4 lines of symmetry, we will produce the table of operations below:

↓→	R_0	R_{90}	R_{180}	R_{270}	H	V	D	D'
R_0	R_0	R_{90}	R_{180}	R_{270}	H	V	D	D'
R_{90}	R_{90}	R_{180}	R_{270}	R_0	D	D'	V	H
R_{180}	R_{180}	R_{270}	R_0	R_{90}	V	H	D'	D
R_{270}	R_{270}	R_0	R_{90}	R_{180}	D'	D	H	D
H	H	D'	V	D	R_0	R_{180}	R_{270}	R_{90}
V	V	D	H	D'	R_{180}	R_0	R_{90}	R_{270}
D	D	H	D'	V	R_{90}	R_{270}	R_0	R_{180}
D'	D'	V	D	H	R_{270}	R_{90}	R_{180}	R_0

(Source: Contemporary Abstract Algebra, p. 25)

Fig 3 – D_8, the symmetry group of a square

Basically this table says that R_0 is the original position of the square, so we have done nothing to it. R_{90} is a square that has been rotated 90 degrees clockwise. H is the horizontal line of symmetry of the square, V is the vertical, D is one diagonal and D' is the other diagonal.

The reason why these operations form a group is that if we take the square under one of the operations and apply another operation to it the result will be a configuration fitting one of the eight operations. So all the operations are finite, closed and contained.

Thus the Dihedral group for the symmetries of a triangle is called D_6, because there are 3 rotations and 3 lines of symmetry. That gives 6 possible operations. The dihedral group for the symmetries of a square is called D_8, so you may have guessed where this is all leading to. We can state (without proof) that the dihedral group of a regular polygon with n sides is called D_{2n}.

The Dihedral group and Adinkra symbol symmetries

i) **The Method**

There are many cases in nature where the symmetrical structure of an object are identical to a dihedral group (of order n) or to a subgroup of a dihedral group. This is the case with crystalline objects as well as with certain viruses and protein structures. The example given in Appendix A is a of a snowflake that has the same symmetries as the dihedral group D_{12} (i.e. the symmetries of a regular hexagon).

It is quite well known that symmetrical art often shows properties that can be described by a dihedral group. This is the main reason why I chose to examine the Adinkra symbols to find out if there are particular symmetries that are more common than others.

In order to do this, I examined the index of G. F. K Arthur's book *Cloth as Metaphor* (p 128 – 187) where he lists 719 Adinkra symbols, together with their meanings, proverbs and other associations. This list is perhaps the most comprehensive list of Adinkra symbols in print (to date). In deciding which Adinkra symbols to choose for analysis, I (subjectively) chose those that looked unique since some of the symbols in the index are variations of the same. Out of the 719 symbols I sampled 84 of them, which produced a sample of approximately 11.68% of all the symbols listed. The result of the analysis is shown in the table below, while the more extensive list can be found in Appendix C.

Refl/Rot	0	2	3	4	>4
0	2	8		1	
1	14				
2		39		1	
3			5		
4				9	
>4					5

Fig 4 – Adinkra symmetries. Summary of sample of 84 symbols

Looking at Appendix A will give some idea of how some Adinkra symbols can be analysed to obtain their symmetries.

i) **Explanation of Results**

The table in figure 4 (above) shows the reflection and rotational symmetries of each symbol analysed (see Appendix A for how the symbols were analysed). What figure 4 shows is that there are many different kinds of Adinkra symbol symmetries. Some shapes have no reflection or rotational symmetries while others have (potentially) infinite reflection and (potentially) infinite rotational symmetries (see numbers 5 and 24 in Appendix C for examples of these).

From the sample of analysed symbols, 5 Adinkra symbols have symmetries that are identical to the dihedral group D_6 (symmetries of a triangle) and 9 symbols have symmetries identical to the dihedral group D_8. 5 symbols had symmetries that had more than 4 lines of symmetry and more than 4 rotational symmetries (see numbers 8, 20 and 60 in Appendix C). These had symmetries identical to dihedral groups D_{16} and D_{32}.

There were also shapes that had only one line of reflection (i.e either horizontal or vertical) or those that had 2 rotational symmetries. Appendix C has more examples of these.

But by far the most common were symbols that had 2 lines of symmetry (horizontal and vertical) and two rotational symmetries. Shapes of this nature can be seen as “one figure made up of two identical parts”. The identical parts are mirror images of each other. I think this is very interesting if you think in terms of an image and its shadow or its mirror.

All the Adinkra symbols that fit this 'most common pattern' fall under dihedral group D4. They are a subgroup of D_4 without all the transformations (i.e. no diagonal reflections and two less rotations).

Finally it should perhaps be mentioned that that dihedral group D_n is a subgroup of dihedral group D_{2n}, so that D_4 is a subgroup of D_8 and D_8 is a subgroup of D_{16}, so that D_4 is also a subgroup of D_{16}. Since the first regular 2-D shape we can have is a a triangle, dihedral groups start at D_6 and continue on to D_8, D_{10} etc, for regular polygons of size 3, 4, 5,...

Where Mathematics meets Metaphysics

Much as the mathematical symmetries of the Adinkra symbols are interesting just by themselves, I believe that at least some of the Adinkra symbols encode vibrational energy patterns. This belief stems from what I have observed in some Native Indian sand paintings, as well as Hindu Indian yantras and Tibetan mandalas. Some of the Adinkra symbols are actually (simplified) carbon copies of patterns found in some sand paintings, mandalas and yantras.

I believe that it may be possible to work out the relationship between the external symmetries of some Adinkra symbols and their energetic-vibrational qualities. Adinkra symbols may be associated with particular colours, numbers and sound patterns and hence may be used in a kabbalistic way.

In order to investigate the vibrational energies underlying some Adinkra symbol symmetries, I used an amethyst pendulum to dowse selected Adinkra symbols. I observed that four movements manifested: west-east, north-south, circular (clockwise) and circular (counter clockwise). Further work is being done in this regard to determine whether it is possible to come up with a schema showing Adinkra symbols with various adjoining qualities. The symmetrical properties of these symbols may provide a helping hand in figuring out this schema.

The author of this article hopes that other spiritually oriented investigators interested in Adinkra may conduct experiments and investigations of their own and share their results. Of particular interest is what other individuals experience (sound, colour, movement) when they meditate on or visualize particular Adinkra symbols.

Conclusion

Although it has emerged from this investigation that the most common symmetry for (symmetrical) Adinkra symbols is two lines of reflection and two rotations (to bring the shape back to its original position), the story about the mathematical and possible metaphysical properties of Adinkra symbols is not over. In fact it is only beginning! It is my hope that more investigations will occur surrounding these symbols in order to learn more about them.

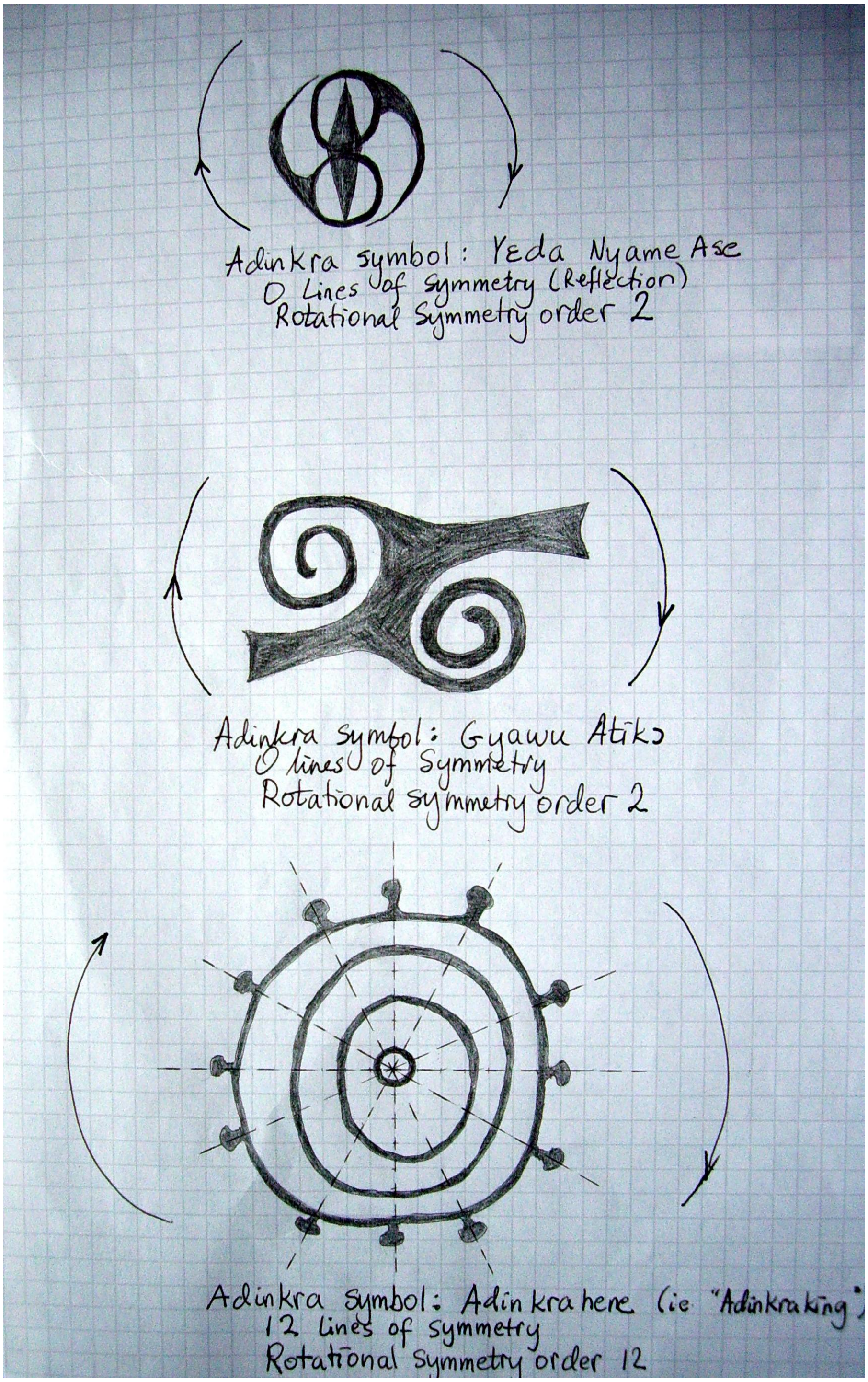
REFERENCES

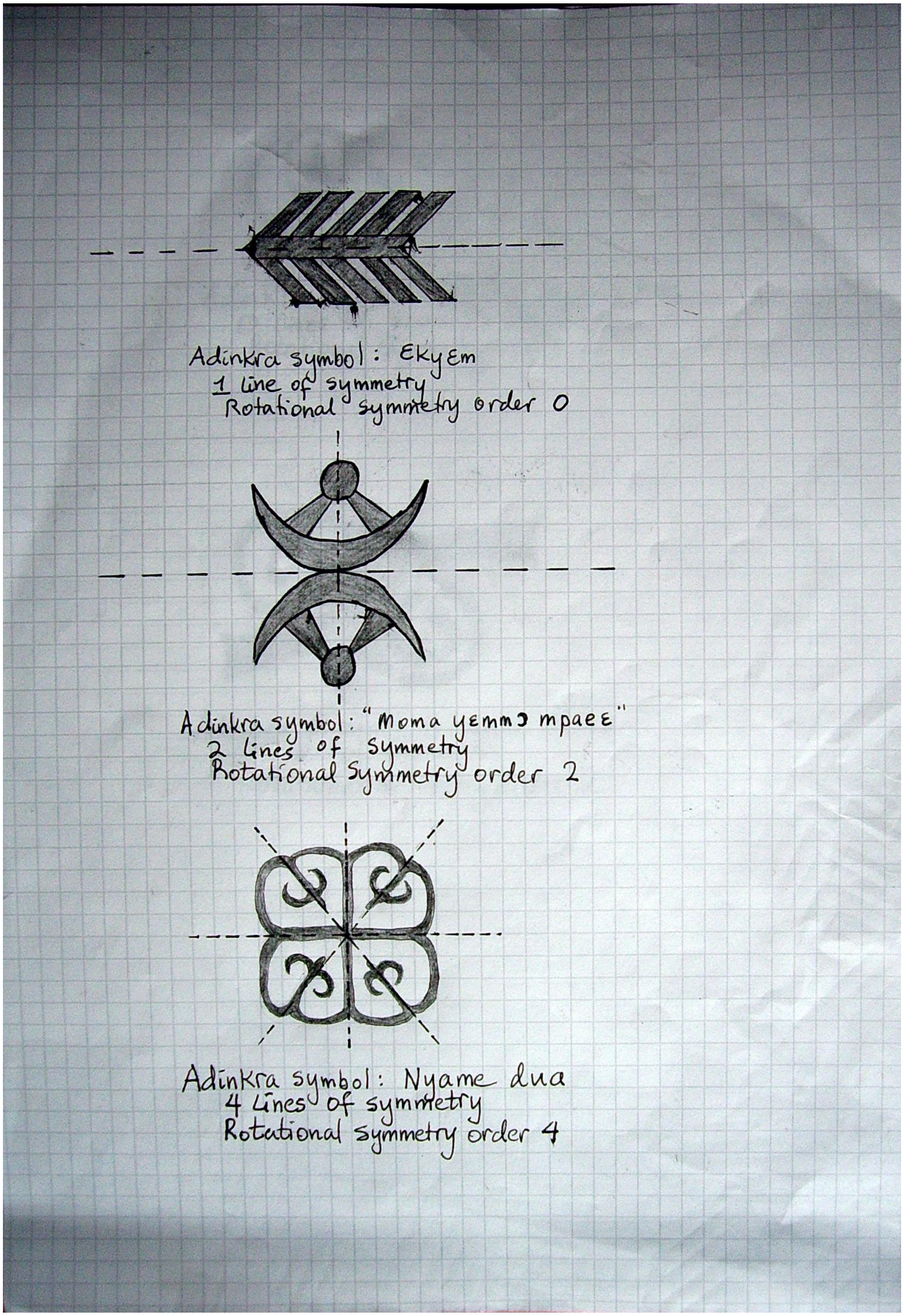
G. F. K. Arthur, *Cloth as Metaphor: (Re)-reading the Adinkra Cloth Symbols of the Akan of Ghana*, Af-oaks Printing Press: 1999

J. A. Gallian, *Contemporary Abstract Algebra*, D. C. Heath and Company: 1990

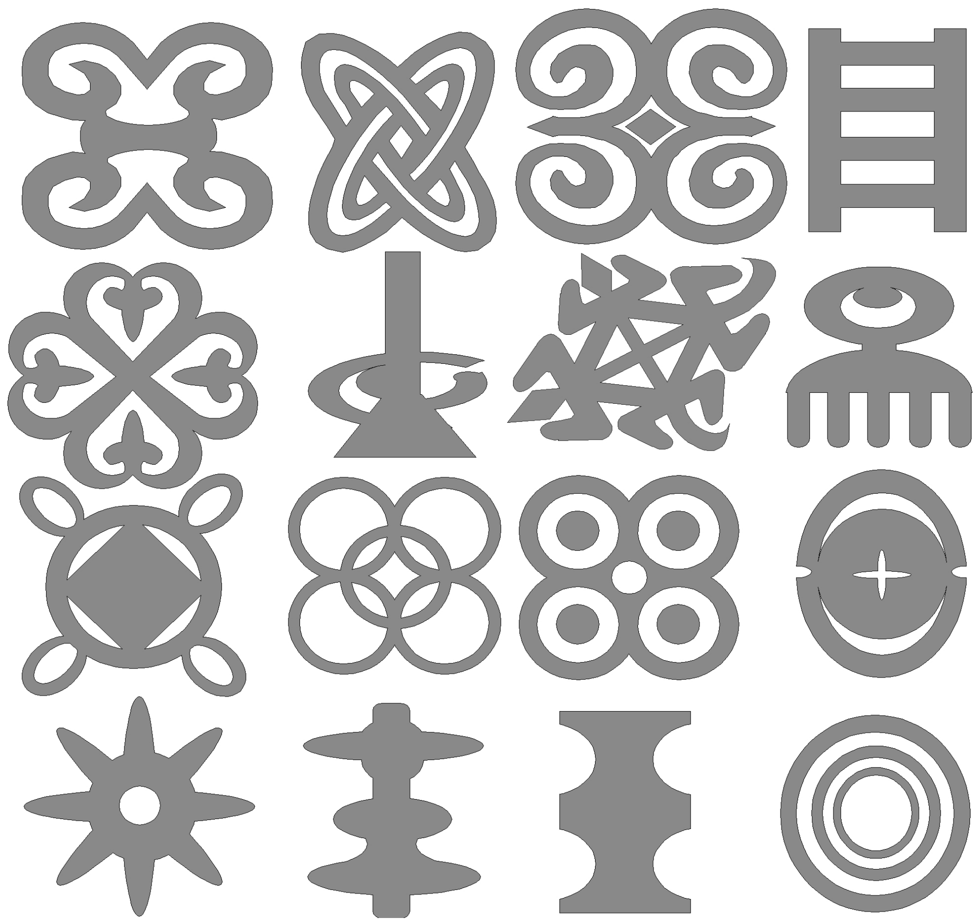
R. S. Rattray, *Religion and Art in Ashanti*, Clarendon Press, Oxford:1927

Appendix A – Examples of Adinkra Symbol Symmetries




Adinkra symbol: Ekyɛm
1 line of symmetry
Rotational symmetry order 0
Adinkra symbol: "Moma yɛmmɔ mpaeɛ"
2 lines of symmetry
Rotational Symmetry order 2
Adinkra symbol: Nyame dua
4 Lines of symmetry
Rotational symmetry order 4

Appendix B – Examples of Adinkra Symbols



(Source: Internet)

An Adinkra Symbol, a snowflake and a Navajo Sand painting
(Source: Internet)

Appendix C – Adinkra Symmetry details

* Refer to Adinkra Index of book *Cloth as Metaphor* to see which Adinkra symbols were used. The numbers in parentheses correspond to the symbol number in the index of symbols.

Number	**Adinkra Name**	**Rotational Symmetry**	**Reflection Symmetry**
1	Abode Santann (1)	0	1
2	Gye Nyame (2)	2	0
3	Odomankoma (13)	3	3
4	Soro ne Asase (14)	2	0
5	Puru (circle) (15)	infinite	infinite
6	Hann ne sum (18)	4	4
7	Asase ye duru (25, 26)	2	2
8	Ananse Ntotan (34)	8	8
9	Sunsum (35)	4	4
10	Nsu (40)	0	1
11	Hye Anhye (42)	2	2
12	Nyame Dua (53)	4	4
13	Kerapa (61)	4	2
14	Momma ye mmbo mpae (70)	2	2
15	Nkrabea (75)	2	0
16	Owuo Atwede (90)	2	2
17	Owuo Mpe Sika (98)	2	2
18	Mema wo hyeden (105)	2	2
19	Woye hwan? (111)	2	2
20	Nsoromma (116)	8	8
21	Biribi wo soro (126)	2	2
22	Ohene Adwa (151)	0	1
23	Sumpie (174)	3	3
24	Adinkra hene (178-185)	Potentially infinite	Potentially infinte

25	Aban pa (193)	2	2
26	Nkuruma kese (195)	4	4
27	Dwennimen (212)	2	2
28	Esono Anantam (222)	4	4
29	Nkotimsefo Pua (234)	4	0
30	Gyawu Atiko (247)	2	0
31	Akoben (253)	0	1
32	Ekyem (269)	0	1
33	Obi nka obi (273)	2	0
34	Dwan Tire (281)	0	1
35	Mmara Krado (290)	2	2
36	Epa (296)	2	2
37	Sepo (299)	0	1
38	Nkonsonkonson (304)	2	2
39	Ese ne tekrema (322)	2	2
40	Pempasie (323)	2	2
41	Boa me (331)	2	2
42	Boa w'aban (336)	2	2
43	Fawohodie (339)	2	2
44	Dua koro (343)	0	1
45	Adwo (349)	2	2
46	Mpaboa (353)	2	2
47	Asase Aban (354)	3	3
48	Asomdwoe	2	2
49	Asambo (375)	2	2
50	Mmofra Banyini (421)	2	2
51	Akoko nan (424)	0	1
52	Abusua do funu (456)	2	2
53	Ye papa (463)	2	2
54	Bu wo ho (464)	2	2
55	Nni awu (478)	0	0

56	Kata wo de so (485)	2	2
57	Enni nsekuro (493)	0	1
58	Wawa aba (506)	2	2
59	Aya (519)	0	1
60	W'ano pe asem (524)	16	16
61	Aka m'eni (530)	0	1
62	Mekyia wo (531)	2	0
63	Saa? (536)	2	2
64	Nantie yie (542)	2	2
65	Obra ye bona (547)	3	3
66	Adasa pe mmoboro (548)	0	1
67	Hwe yie (554)	2	2
68	Bo wo ho ban (555)	2	2
69	Hwe w'akan mu yie (556)	2	0
70	Mensuro wo (559)	0	0
71	Dwen wo ho (563)	2	2
72	Bese saka (576)	4	4
73	Asetena pa (604)	2	2
74	Sankofa (642)	0	1
75	Mate Masie (652)	4	4
76	Kuntankantan (666)	4	4
77	Nkore (670)	2	2
78	Gyina Pintinn (672)	2	2
79	Dwen hwe kan (673)	2	2
80	Wudu nkwanta a (683)	2	2
81	Aburuburo kosua (710)	2	0
82	Ananse anton kasa (715)	2	2
83	Ahinansa (717)	3	3
84	Nkabom (719)	2	0